## Peak Debt

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Consumer spending now accounts for about $70 \%$ of economic activity in the United States. Constraints on this spending are of great interest because they put limits on the potential for future growth in the overall economy. From the mid1960's for a period of almost 30 years consumer purchasing power, defined as the ratio of consumer spending to after-tax personal income, was very nearly constant. Beginning in the mid-1990's, however, something significant happened that changed people's spending habits. Consumers began both to save less and to take on more debt than ever before to finance personal expenditures. This is now a well known story - although for most of the population real (inflation adjusted) income has not risen since 1970, over the last decade relaxed lending standards and the invention of new credit instruments have made it much easier to obtain mortgages, credit cards, and loans of all kinds.

Looking forward, it's natural to ask how long might this situation persist? We already see rising default rates on mortgages and credit cards; and the crunch brought on by falling house prices has made it much more difficult to obtain credit of any sort. We would like to consider the idea that personal debt can be treated as if it is something like a finite resource. What are the consequences that a ceiling on the total amount of debt that can be supported by borrowers would have for consumer spending, and by extension, for the growth of the economy as a whole?

In this analysis we will make use of the National Income and Product Accounts (NIPA) quarterly data series that are published by the Bureau of Economic Analysis [1]. The NIPA's provide uniform measures of aggregate U.S. economic activity extending over many decades, and include gross domestic product (GDP), gross domestic income (GDI), personal income, and personal savings.

We can write a simple self-consistent expression for consumer spending, P , as income, minus savings, minus current debt service, plus new borrowing [2]:

$$
\mathrm{P}=\mathrm{I}-\mathrm{S}-\left(\mathrm{i}_{\mathrm{eff}} \cdot \mathrm{~A}\right)+\mathrm{dA} / \mathrm{dt}
$$

where $I$ is disposable (after-tax) personal income, $S$ is personal savings, $A$ is the total accumulated debt at time $t$, and $i_{\text {eff }}$ is the effective interest rate being paid to service that debt. New borrowing is just the change in the total debt, dA/dt. We will consider each of these contributing factors in turn.

For most of the population, real (inflation adjusted) income has not changed since 1970. This can be seen from Figure 1 which shows percentile distributions of real average incomes including capital gains through 2005 [3]. While it is true


Figure 1. Percentile distributions of real average incomes including capital gains through 2005. Solid lines: linear regressions (LR) 1970-2005; dotted line: LR 1970-1995.
that in the late 1990's the aggregate average income, $\Pi(0-100)$, did increase by a small amount over the longer-term effectively-flat trend of $0.06 \%$ per year, all of this gain was confined to the top one percent of the population. The average income for the bottom $99 \%, \Pi(0-99)$, has been essentially constant at $-0.01 \%$ per year for almost 40 years. The bottom $95 \%, \Pi(0-95)$, actually declined by $-0.21 \%$ per year; and the bottom $90 \%, \Pi(0-90)$, by $-0.34 \%$ per year over this period.

The implication of nominally flat real income is that if we look at the ratio (P/I) of consumer spending to disposable personal income, the obscuring effects of inflation will be minimized, and we will have a good measure of real purchasing power. Normalizing equation $\{1\}$ to I we have:

$$
(\mathrm{P} / \mathrm{I})=1-(\mathrm{S} / \mathrm{I})-\mathrm{i}_{\mathrm{eff}} \cdot(\mathrm{~A} / \mathrm{I})+(\mathrm{dA} / \mathrm{dt}) / \mathrm{I}
$$

Quarterly spending data for (P/I) from 1965 to 2008 are plotted in Figure 2, and the respective savings data for (S/I) in Figure 3 [1]. With regard to the aggregate effective interest on debt, $i_{\text {eff }}$, we note that mortgage, credit card, and many other lending rates tend to scale with the returns on long-term treasuries usually those of the ten year bill. Constant-maturity rates for the ten year treasury, $\mathrm{i}_{10 \mathrm{~T}}$, are plotted in Figure 4 [4]; and we will make the reasonable assumption that $\mathrm{i}_{\mathrm{eff}}=\delta \cdot \mathrm{i}_{10 \mathrm{~T}}$, where $\delta \sim \mathrm{O}(1)$ is a constant to be determined.

We see from the figures that spending, (P/I), was flat from the mid 1960's to the early 1990's after which it began to grow steadily to the present. The corresponding savings, (S/I), persisted at the $9-10 \%$ level through the late 1980's and then dropped smoothly to less than $1 \%$ in recent times. Interestingly, the period of flat spending and savings occurred in an environment of rising interest rates. When interest rates began their long-term decline, savings rates also


Figure 2. Consumer spending (P/I).


Figure 3. Personal savings (S/I).


Figure 4. Interest rates on 10-year treasuries $\mathrm{i}_{10 \mathrm{~T}}$.


Figure 5. Total consumer debt (A/I).
started to fall, and spending - abetted by both lower savings rates and lower interest rates on debt - began to rise.

Of the terms in equation $\{2\}$, it remains for us to examine the time evolution of the accumulated total debt, (A/I). We note that the equation can be rewritten:

$$
\left.(\mathrm{A} / \mathrm{I})-(\mathrm{dA} / \mathrm{dt}) /\left(\mathrm{i}_{\mathrm{eff}} \cdot \mathrm{I}\right)\right)=[1-(\mathrm{P} / \mathrm{I})-(\mathrm{S} / \mathrm{I})] / \mathrm{i}_{\mathrm{eff}} .
$$

Because $A$ is larger than $\mathrm{dA} / \mathrm{dt}$ by about two orders of magnitude, the righthand side of equation $\{3\}$ gives a good estimate of the total normalized debt:

$$
(\mathrm{A} / \mathrm{I}) \sim[1-(\mathrm{P} / \mathrm{I})-(\mathrm{S} / \mathrm{I})] / \mathrm{i}_{\mathrm{eff}}
$$

This expression can be evaluated and is plotted in Figure 5. We see that, as was the case for ( $\mathrm{P} / \mathrm{I}$ ) and ( $\mathrm{S} / \mathrm{I}$ ), the normalized total consumer debt ( $\mathrm{A} / \mathrm{I}$ ) changed little from 1965 to 1990. It then began to increase steadily with the secular decline in interest rates.

We want to investigate the situation in which the new debt initiated in response to the changing credit conditions after 1990 rises from a flat background level $\left(A_{O} / I\right)$ to some new maximum value $\left(\left(A_{O}+A_{M}\right) / I\right)$. This new debt ceiling can be understood to come about from a combination of the consumer's eventual inability to support any additional borrowing, and the lender's growing unwillingness to take on the risk of increasingly more marginal loans. The form of the curve of rising debt in Figure 5 suggests that the shape of the timedependent accumulation of new debt might be well described by the same logistic growth equations that are often used to model the evolution of bounded populations or the depletion of natural resources such as oil.

Typical logistic growth is illustrated in Figure 6. In this case, a quantity B grows from a background level $B_{O}$ to a maximum $\left(B_{O}+B_{M}\right)$ over a period of time determined by the growth rate parameter $R_{\text {eff }}$. The rate of growth in $B, d B / d t$,


Figure 6. Example of logistic growth.
is most rapid at time $t_{1 / 2}$. A key feature of monotonic growth from one level to another is that the rate of growth in $B$ peaks at $t_{1 / 2}$, long before $B$ itself reaches its maximum value. A pertinent example that supports the use of this model for debt accumulation is the logistic growth of personal debt in the period leading up to the Great Depression of the 1930's, which is discussed in Appendix A.

Within the framework of logistic debt growth we can write:

$$
(\mathrm{A} / \mathrm{I})=(1 / \varepsilon) \cdot\left(\mathrm{A}_{\mathrm{M}} / \mathrm{I}\right)+\left(\mathrm{A}_{\mathrm{O}} / \mathrm{I}\right)
$$

and

$$
(\mathrm{dA} / \mathrm{dt}) / \mathrm{I}=\left(\mathrm{A}_{\mathrm{M}} / \mathrm{I}\right) \cdot \mathrm{r}_{\mathrm{eff}} \cdot(\varepsilon-1) / \varepsilon^{2}
$$

where

$$
\varepsilon(\mathrm{t})=\left\{1+\exp \left[\mathrm{r}_{\mathrm{eff}} \cdot\left(\mathrm{t}_{1 / 2}-\mathrm{t}\right)\right]\right\}
$$

The effective rate of debt accumulation is governed by $\mathrm{r}_{\text {eff }}$, and the peak in borrowing occurs at time $t_{1 / 2}$. We now have from Equation $\{2\}$ :

$$
(\mathrm{P} / \mathrm{I})=\left\{1-(\mathrm{S} / \mathrm{I})-\left(\mathrm{A}_{\mathrm{M}} / \mathrm{I}\right) \cdot\left[\mathrm{i}_{\mathrm{eff}} \cdot\left[(1 / \varepsilon)+\left(\mathrm{A}_{\mathrm{O}} / \mathrm{I}\right) /\left(\mathrm{A}_{\mathrm{M}} / \mathrm{I}\right)\right]-\mathrm{r}_{\mathrm{eff}} \cdot(\varepsilon-1) / \varepsilon^{2}\right]\right.
$$

Cast in this form, (P/I) depends on five parameters that are nominally constant over time: $\left(\mathrm{A}_{\mathrm{O}} / \mathrm{I}\right),\left(\mathrm{A}_{\mathrm{M}} / \mathrm{I}\right), \mathrm{r}_{\text {eff }}, \mathrm{t}_{1 / 2}$, and $\delta$, where $\mathrm{i}_{\text {eff }}=\delta \cdot \mathrm{i}_{10 \mathrm{~T}}$. The background debt level $\left(A_{0} / I\right)$ is seen to be reasonably time independent from the flat tail in Figure 5; and the quantities $\left(A_{M} / I\right), r_{e f f}$, and $t_{1 / 2}$ arise as constants of the logistic growth formulation itself.

With an established functional form for consumer purchasing power, (P/I), we can proceed with least-squares fits of the data of Figures 2, 3, and 4 to determine


Figure 7. Best fit of purchasing power (equation $\{8\}$ ).


Figure 8. Best fit of the logistic debt functions (equation $\{8\}$ ).
values for the set of free parameters. The best-fit curve for (P/I) is plotted in Figure 7, and the corresponding logistic debt growth functions, (A/I) and $(\mathrm{dA} / \mathrm{dt}) / \mathrm{I}$, are shown in Figure 8. A wide range of initial configurations was examined to insure that an absolute minimum in $\mathrm{X}^{2}$ was obtained over-all.

We see from Figure 7 that our expression for ( $\mathrm{P} / \mathrm{I}$ ) does a rather good job of matching the nuances of the data series. The curve suggests that purchasing power has leveled off and may in fact no longer be increasing. The shape of the rise in total debt shown in Figure 8 is also very well fit by the logistic growth equation (see also Appendix A). The rate-of-change in the logistic growth curve implies that new borrowing probably reached its peak in 2005 and is now in decline. We will refer to the peak in the rate of accumulation of new debt as Peak Debt, in conformity with the popular use of the designation "Peak Oil" to refer to the maximum in the rate of petroleum production.

We want to consider the consequences that Peak Debt will have for the future of consumer purchasing power. Predictions about the behavior of $(P / I)$ from


Figure 9. Baseline projection: interest and savings rates held constant at current levels.


Figure 10. Baseline (P/I) prediction corresponding to the projections of Figure 9.
equation $\{8\}$ depend on projections of both interest and savings rates. In a sense we are fortunate that both (S/I) and $\mathrm{i}_{10 \mathrm{~T}}$ are now at historically low levels, and that it is very unlikely that either can drop much below its current value or remain where it is indefinitely. Consequently, it is informative to consider as a limiting case the very optimistic assumptions, illustrated in Figure 9, that both (S/I) and $\mathrm{i}_{10 \mathrm{~T}}$ will stay fixed at their current low values over the next several decades. We can regard this optimistic scenario as a baseline or reference, and proceed with the evaluation of equation $\{8\}$ to obtain the prediction for (P/I) shown in Figure 10. In this case, we see that purchasing power has already reached a maximum and that it will begin to decline and then level off about 5\% below its present value. At best, this prediction would mean that the consumers' contribution to economic growth is over. The shape and magnitude of the decline in this baseline projection of ( $\mathrm{P} / \mathrm{I}$ ) closely resembles that seen in the period from the late 1960's to the early 1980's when the economy as a whole behaved badly.

More realistically, we know that rising inflation, the falling dollar, the trade deficit, and rising energy costs - all of which are closely interconnected - argue that interest rates will have to rise from their present almost unprecedentedly


Figure 11. Projected interest rates rise to $6 \%$, and savings rates rise to $3 \%$.


Figure 12. (P/I) prediction corresponding to the projections of Figure 11.
low values. We will make the two very conservative assumptions illustrated in Figure 11: (1) that interest rates will follow their historic trajectory upward, but only rise to maximum of $6 \%$ on the 10-year treasury (i.e. $3 \%$ above a nominal long-term inflation rate of $3 \%$, and less than half of the peak seen in the early 1980's); and (2) that, prompted by rising interest rates and economic distress, savings will increase as they fell, but only to a third of the rate seen in the 1970's. The corresponding prediction for $(\mathrm{P} / \mathrm{I})$ is shown in Figure 12. In this case, we see a very substantial drop in purchasing power that almost mirrors the rise that occurred after 1990. This degree of decline in (P/I) would certainly signal a long period of negative economic growth and probably a general depression.

It might be argued in objection to these results, that the changes in the nature and availability of credit that we are already seeing may once again alter the circumstances of consumer spending and perhaps in some way ameliorate the problem of declining purchasing power. Because the data presented in Figure 1 suggest that it is quite unrealistic to expect significant growth in real personal income, there are really only two plausible ways by which the current large overhang of debt might be reduced: through default or through hyper-inflation. We will consider the impact of each of these on consumer spending in turn.


Figure $13.20 \%$ of outstanding debt defaults, and new borrowing reduced in proportion.


Figure 14. (P/I) prediction corresponding to the projections of Figures $11 \& 13$.
Significant rates of default and bankruptcy would have a large negative impact on the financial system as a whole. Aside from this concern, (P/I) would be affected in three main ways: there would be less debt service to be deducted from income, but interest rates would rise, and it would be very hard for defaulters to get additional loans. If we assume, as is shown in Figure 13, that $20 \%$ of all outstanding debt goes into default and that new borrowing is reduced in proportion, we see from Figure 14 that there would be in fact little overall change in the predicted decline in consumer spending from what was found for the no-default case of Figure 12.

Nominal inflation is included in our analysis by considering only the ratios of quantities-of-interest to personal income. Under hyper-inflation, incomes will lag both prices and the cost of new debt, but will appear to rise relative to extant long-term debt. The two contrary effects will tend to cancel each other, and the ratios-to-I will probably still remain a reasonable overall compensation for inflation. This was just the situation during the hyper-inflation that occurred between 1970 and the early 1980's. But hyper-inflation also leads to significant


Figure 15. Interest rates return to historic maxima with continuing low savings rates.


Figure 16. (P/I) prediction corresponding to the projections of Figure 15.
increases in real interest rates. If inflation were again allowed to get out of hand as it did in the 1970's, interest rates could return to the historic levels shown in Figure 15. In this case, even if it is assumed that there would be no increase in savings in response to the rising interest rates and economic uncertainty, the drop in purchasing power would be dramatic. Figure 16 shows that (P/I) would fall from its peak value by about $20 \%$. If savings rates also returned to their 1970's level, the decline from the peak would be by more than $30 \%$. In either case, the consequences for the broader economy would be correspondingly grim.

In general, the prospects for consumer spending predicted by our analysis are rather bleak. How critical to these conclusions are the specific details of our Peak-Debt formulation in terms of the logistic growth functions shown in Figure 8? From equation $\{1\}$ we know that growth in spending depends on the rate of new borrowing being able to over-compensate for the cost of servicing the total extant debt. This means that when ( $\mathrm{dA} / \mathrm{dt}$ ) is no longer increasing, the growing debt service on A will eventually come to dominate. As an example, we can consider the limiting case, illustrated in Figure 17, in which (A/I) continues to increase indefinitely - the actual ceiling on the total debt is left indeterminate. The fact that the forward projection of (A/I) is linear means that


Figure 17. Linear extrapolation of (A/I) to an indeterminate debt ceiling.


Figure 18. ( $\mathrm{P} / \mathrm{I}$ ) prediction corresponding to the projections of Figures $11 \& 17$.
$(\mathrm{dA} / \mathrm{dt}) / \mathrm{I}$ has already grown to a maximum value (that of the slope of the (A/I)-line). The corresponding prediction for (P/I), with the rate projections of Figure 11, is shown in Figure 18. We see that the decline in consumer purchasing power is even greater than was seen in Figure 12. This result can be understood in that even more total debt accumulates under the conditions of this rather unrealistic limiting-case example than it does when the best-fit of the logisticgrowth equation is used.

In conclusion, we see that the application of a rather simple model for the dependence of consumer purchasing power on the conditions and availability of credit seems to indicate that there will be no easy resolution to the consequences of reaching a peak in the rate at which new debt can be assumed (Peak Debt). The model itself depends only on the self-consistent set of definitions of the quantities involved, and is not particularly sensitive to the detailed way in which debt increases. Every plausible scenario that we have considered points to a significant decline in consumer spending in the near future. Eliminating part of the accumulated debt through bankruptcy and default will not change the
fundamental picture with regard to the decline in purchasing power. Nor will permitting hyper-inflation to develop. A significant devaluation of the currency would only exacerbate the inevitable unwinding.
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## References

[1] U.S. Department of Commerce: Bureau of Economic Analysis, April 2008.
[2] The BEA defines its various tabulated NIPA Data Series as follows:
Personal Income (PI): Income from all sources
Disposable Personal Income (DPI):
Personal Consumption (PCEC):
(PI) - (all taxes)
All purchased goods and services
Personal Outlays (PO):
(PCEC) + (interest) + (transfers)
Personal Savings (PSAVE):
(DPI) - (PO)
Thus: $\quad(\mathrm{DPI})=(\mathrm{PO})+(\mathrm{PSAVE})$,
or

$$
(\text { PCEC })=(\mathrm{DPI})-(\text { PSAVE })-(\text { interest payments }) \pm(\text { transfers out } / \mathrm{in})
$$

It should be noted that this equation arises simply from the self-consistent set of definitions of the quantities involved.

If $A$ is total consumer debt, our corresponding notation is:
$\mathrm{I}=(\mathrm{DPI})$
$\mathrm{P}=(\mathrm{PCEC})$
S = (PSAVE)
$\mathrm{i}_{\text {eff }} \cdot \mathrm{A}=$ (interest payments)
$\mathrm{dA} / \mathrm{dt}=$ current borrowing $\quad$ (i.e. transfers-in)
and we have, from the definitional equation above, our equation $\{1\}$ :

$$
\mathrm{P}=\mathrm{I}-\mathrm{S}-\mathrm{i}_{\mathrm{eff}} \cdot \mathrm{~A}+\mathrm{dA} / \mathrm{dt}
$$

[3] T. Piketty and E. Saez. Income inequality in the United States 1913-1998. Quarterly Journal of Economics 118(1), 1-39 (2003). Updated November 2004 (Preprint).
Updated Tables 2005. Available online (2006) at:
http: / / emlab.berkeley.edu / users / saez / piketty-saezOUP04US.pdf
[4] Ten year Treasury constant maturity rate, BEA Series GS10, (2008).
[5] David C. Wheelock. The Federal Response to Home Mortgage Distress: Lessons from the Great Depression. Federal Reserve Bank of St. Louis Review, May/June 2008, 90(3, Part 1), pp. 133-48.

## Appendix A: Peak Debt and the Great Depression.

The features of recent economic history, including the introduction of new instruments of credit, low interest rates, easy loans, and wide-spread speculation, have much in common with the situation found during the 1920's. How did debt accumulate in the period prior to the onset of the great depression? Because residential mortgage debt typically accounts for about two-thirds of total personal debt, we can view the former as a reasonable proxy for the latter. The data points in Figure A1 show the real total residential mortgage debt recorded annually between 1913 and 1940 [5]. That the distribution provides a classic example of logistic growth is demonstrated by the excellent fit of the red curve $(M)$. The derivative of this curve ( $\mathrm{dM} / \mathrm{dt}$ ) is shown in blue and indicates that the peak in borrowing occurred in 1925. The depression began about five years later.


Figure A1. Total real residential mortgage debt from 1913 to 1940.
This history can perhaps guide the interpretation of our analysis of the present peak-debt situation shown in Figure 8. We note that the current run-up of debt seems to be happening about three times more slowly than did the one in the 1920's. A naive analogy drawn between the two cases might suggest that there could be of order a decade remaining before a serious collapse would be expected to occur. However, as is discussed in the main text of this document, the actual time frame in which these economic events will play themselves out will depend strongly on the evolution of interest rates, savings rates, and government policies that affect inflation and the value of the currency. The primary implication of our analysis remains and is re-enforced: there appears to be no way to avoid the dire economic consequences of an over-accumulation of personal debt.

